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Energy Technology Assessment

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Energy Economics

Chapter 1:

Energy Technology Assessment

Abstract

The aim of this paper seeks to introduce the basis of the energy economics models defined as a market equilibrium problems-mixed complementarily problem (MCP). This technique allows the integration of bottom-up programming models of the energy system into top-down general computable equilibrium models (CGE) of the overall economy. A complementarily scheme involves both primal and dual relationships, often doubling the number of equations and the scope of error. When the underlying optimization includes upper and lower bounds (many decision variables), the explicit treatment of associated income effects may become very complex. A convenient MCP formulation of both, top-down & bottom-up energy system models for energy policy analysis requires the uses of complementary methods to solve the economic equilibrium.

1. Introduction

The two modelling paradigms in order to represent interactions between the energy system and the economy are the Top-down & Bottom-up models (Hourcade et al., 2006) – respectively these terms regards aggregate and disaggregated models. The first one emphasizes economy-wide features, while those in the second one focus on sectoral and technological details. The dichotomy of energy-economy models into these two categories is sometimes traced back to competing paradigms (Weyant, 1985).

Top-down models examine the broader economy and incorporate feedback effects between different markets triggered by policy-induced changes in relative prices and incomes. They typically *do not provide technological details of energy production or conversion* (e.g., non incorporate assumptions about discrete energy technologies and how costs will evolve in the future; they also violate fundamental physical restrictions of thermodynamic). Energy sectors and other non-energy sectors are mostly aggregated by production functions which capture substitution (transformation) possibilities via substitution (transformation) elasticities.

Bottom-up models are usually defined as mathematical programming problems (describe current and prospective technologies in detail). They are well suited in order to analyse specific technological changes or policies regarding efficiency (productivity) standards. Beyond the lack of economy-wide interactions, a common shortcoming of the bottom-up approach emerges from the integrability conditions of mathematical programs (Pressman, 1970; Takayma & Judge, 1971). Since first-order conditions impose efficient allocation, primal or dual mathematical programs fail to account for initial second best (e.g., initial tax distortions or market failures).

There are several hybrid schemes that try to combine modelling efforts of both approaches - broadly classified into three followings categories:

The first one attempts to couple *existing* large-scale bottom-up and top-down models (e.g., Hofman & Jorgenson, 1976; Hogan & Weyant, 1982; Messner & Strubegger, 1987; Drouet et al., 2005; Schäfer & Jacoby 2006). Due to the heterogeneity in complexity and accounting methods across the sub-models such a “soft-link” approach may face substantial problems in achieving overall consistency and convergence of iterative solution algorithms.

The second category focuses on one model type complemented by reduced form representation of the other. A common approach within this category is the linkage of bottom-up energy system models with a highly aggregate one-sector representation of macro-economic production and consumption in a single optimization framework (e.g. Manne, 1977; Manne et al., 2006; Bahn et al.,1999; Messner & Schrattenholzer, 2000; Bosetti et al., 2006).

The third more recent category combines bottom-up and top-down characteristics directly through the specification of market equilibrium models as Mixed Complementarily Problems (Cottle & Pang, 1992; Rutherford, 1995). The explicit representation of weak inequalities and complementarities between decision variables and market equilibrium conditions in the MCP formulation permits the modeller to capture both, technological details and economic richness in a single mathematical format (Böhringer, 1998; Böhringer and Rutherford, 2007). The availability of robust large-scale solvers for MCP problems (Dirkse & Ferris, 1995) has promoted the implementation of hybrid energy-economy models in the MCP format to analyse energy regulation policies (e.g., Böhringer et al., 2003; Frei et al., 2003).

Despite the appeal of the integrated MCP approach regarding to flexibility and overall consistency, complexity and dimensionality may impose significant restrictions to their application (optimization problem includes many upper and lower bounds). Bounds can be incorporated in the MCP framework but the explicit representation of associated income effects may become intractable.

An integrated MCP model can be decomposed and solved iteratively: some complementarily methods are used to solve the top-down economic equilibrium model and quadratic programming is also applied to solve the underlying bottom-up energy (supply) model. Rapid convergence of iterative procedure (e.g., Jacobi algorithm) requires that the decomposed energy sector be small in value terms relative to the rest of the economy - Marshallian demand approximation in the energy sector model provides a precise local representation of the general equilibrium demand.

The combination of both approaches constitutes a long-standing challenge in applied energy policy analysis. The formulation of economic equilibrium conditions as some mixed complementarily problem provides a unifying framework for combining technological details and economic richness.

In order to propose a decomposition procedure that overcomes the limitations of the integrated mixed complementarily approach is possible to combine different mathematical formats (e.g., mixed complementarily and mathematical programming). Complementarily methods will fit in order to solve the top-down economic equilibrium model - quadratic programmings are more precise solving underlying bottom-up energy supply model.

2. Integrated Model Formulation

The MCP approach provides a general mathematical format that covers weak inequalities, (i.e. a mixture of equations and inequalities, and complementarily between variables and functional relationships), with linear or non-linear system of equations or mathematical programming. Therefore, the formulation relaxes the integrability constraints for equilibrium conditions, which emerge as first-order conditions from primal or dual optimization problems (Böhringer & Rutherford, 2007) – also allowing the representation of market inefficiencies (e.g., Spillover effect).

One possible formulation of an integrated model considers a competitive (Arrow-Debreu) economy with n commodities (including economic goods, energy goods and primary factors) indexed by i, m production activities (sectors) indexed by j, and h households (including government) indexed by k. If we extend the MCP framework suggested by Mathiesen (1985) embedding an explicit linear-programming sub-model of energy supply in the economy - the decision variables might be classified in the following way:

p Denotes a non-negative n-vector in prices for all goods factors.

y Is a non-negative m-vector for activity levels of constant returns to scale (CRTS) production sector.

M Is an h-vector of consumer income levels.

e Represents a non-negative n-vector of net energy system outputs (e.g. oil, gas, and biomass).

x Denotes a non-negative n-vector of energy system inputs (e.g. labour, capital)

The competitive market equilibrium for this economy must be represented by an economic vector (i.e., activity levels, non-negative vector of prices, and non-negative vector of incomes):

• No production activity makes a positive profit (zero-profit condition):

$$-\Pi_j(p) \geq 0 \quad (1)$$

$\Pi_j(p)$ Denotes the unit profit function for CRTS production activity *j*, which is calculated as the difference between unit revenue and unit cost (i.e., $\Pi_j(p) = r_j(p) - c_j(p)$).

• Excess of supply, (i.e. supply minus demand, is non-negative for all goods and factors) the market clearance condition is:

$$\sum_j \nabla\Pi_j(p) y_j + \sum_k w_k + e \geq \sum_k d_k(p, M_k) + x \quad (2)$$

w_k Is the initial endowment vector for household *k* and $d(k, M_k)$ is the utility-maximizing demand vector for household *k*.

• Expenditure for each household equal's their income (budget constraint):

$$\mathrm{M}_k = p^T\left[w_k + \theta_k(e - x)\right] \quad (3)$$

θ_k, represents the share of energy-sector rents that accrue to household *k* (rents depend on household ownership of energy resources). The consumer income equation is different from the zero profit and market clearing conditions is non explicit complementarily. The income variables will be added to the equilibrium system in order to simplify the equation of household demand. Furthermore, we assume that the equilibrium levels of energy sector outputs and inputs are consistent with profit-maximization, taking market prices as given:

• Energy sector supply and demand vectors are profit-maximizing choices subject to technical constraints. That is, *e* and *x* solve a linear programming model (non-linear). Let us to assume the following bottom-up model:

$$\max p^T(e - x) \quad (4)$$

$$\text{Subject to.}$$

$$\mathrm{A}x + \mathrm{B}z \geq C_e$$

$$e, x \geq 0; l \leq z \leq u$$

$\mathrm{A}, C \in R^{M*n}$, and $B \in R^{M*N}$ characterize technical constraints and $z \in R^N$ denotes decision variables of the energy system.

For example if we assume a linear program the integrated model will be incorporated through the associated Kuhn-Tucker conditions and solved simultaneously with the equilibrium conditions (1)-(3):

$$C^T \pi \geq p; \quad e \geq 0; \quad e^T (C^T \pi - p) = 0$$

$$p \geq A^T \pi; \quad x \geq 0; \quad x^T (p - A^T \pi) = 0$$

$$Ax + Bz \geq Ce; \quad \pi \geq 0; \quad \pi^T (Ax + Bz - Ce) = 0$$

$$l \leq z \leq u; \quad \lambda, \mu \geq 0; \quad \lambda(z - l) = 0; \quad \mu(u - z) = 0$$

$$\lambda + B^T \pi = \mu;$$

The attribution of energy-sector rents to households is the equation (3) rewritten in the following way:

$$M_k = p^T \omega_k + \Theta_k (\mu^T u + \lambda^T l)$$

$\Theta_k \in R^{h*N}$ Determines rents on energy-sector resources to households. The integrated equilibrium for this hybrid model consists in m+3n+h+M+3N equations as compared with the standard economic model of dimension $m + n + h$ and the original linear programming model with M constraints and $N + 2n$ variables.

3. Decomposition

The insertion of the energy-sector sub-model within the general equilibrium framework imply computational challenges (dimensions of energy sectors) – captured by $N + M$. While an integrated MCP formulation is attractive for highly aggregated system (macro energy system) representations, it has limitations for large-scale systems with bounds on many variables - awkward to implement and has difficulties too explain associated income effects.

One interesting possibility is the complementarily of schemes with a decomposition between integrated model in which the energy system bottom-up component will be computed separately from the top-down economic general equilibrium sub-model. The procedure involves iterative solution for the top-down general equilibrium model given net supplies from the bottom-up energy sector sub-model - followed by the solution of the energy sector sub-model based on a locally calibrated set of *demand functions* for energy sector outputs. When $(e - x)$ and θ are given exogenously, the top-down general equilibrium model can be solved as a complementarily problem of dimension $m + n + h$.

Suppose that computed equilibrium prices are $\bar{p}$ (based on an initial estimate for the energy sector response $\bar{e}, \bar{x}$ and $\bar{\theta}$). The next step in a recursive solution procedure updates the values of $(e - x)$ and θ based on $\bar{p}$. One might then consider a direct solution of the profit-maximizing, which characterizes the choices of an individual firm. However, is possible that this approach is quite likely to fail because the profit-maximizing linear program (4) does not properly link to market demand responses regarding changes in energy prices. Suppose that these demand elasticities are given by ε, next we might write the demand for energy good *i* as:

$$e_i(p) = \bar{e}_i\left[1 - \varepsilon_i(p_i / \bar{p}_i - 1)\right]$$

Where ε_i is the demand elasticity and $\bar{e}_i$, $\bar{p}_i$ denote the reference quantities and prices for the demand function calibration. Hence, the calibrated inverse demand function is:

$$p_i(e) = \bar{p}_i\left[1 - (1 - e_i / \bar{e}_i) / \varepsilon_i\right]$$

and the integrated market demand function is:

$$\int p_i(e_i)de_i = \bar{p}_i e_i\left[1 - \frac{e_i - 2\bar{e}_i}{2\varepsilon_i \bar{e}_i}\right];$$

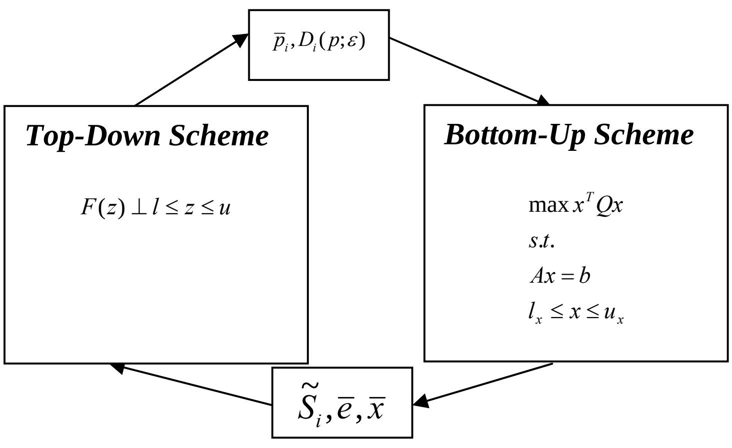
An aggregate (integrated) multi-commodity energy system may then be solved as a quadratic programming problem:

$$\max \bar{p}^T(e - x) - \frac{1}{2}\sum_i \frac{\bar{p}_i e_i}{\varepsilon_i \bar{e}_i}(e_i - 2\bar{e}_i) \quad (5)$$

subject to the same constraints which appear in (4) in order to compute a partial market equilibrium based on linear demand functions locally calibrated to the given macroeconomic equilibrium.

In the Figure 1, we show the basic steps involved in the iterative model solution. The top-down model is solved as a complementarily problem, taking net energy supplies $\bar{e}_i$ and energy sector inputs $\bar{x}$ as given. The computed equilibrium determines prices $\bar{p}_i$ and a set of linear demand curves for energy sector outputs $D_i(p;\varepsilon)$. These demand curves and relative prices parameterize the bottom-up model which may be solved as a quadratic program.

Figure 1: *Iterative Decomposition Algorithm*



We essentially use the decomposition procedure to compute a single sector Marshallian market equilibrium with a nonlinear demand curve DD and a piecewise linear supply schedule S. The starting point of the algorithm is the initial estimate of Q_0 for the quantity of energy supply. This quantity has an associated market price (marginal willingness to pay) p_0 and market equilibrium point *a*. Having computed this equilibrium (ignoring the supply schedule), the algorithm next evaluates the energy market based on a linear demand curve calibrated to the market equilibrium at point *a*. The solution to the supply problem maximizes the sum of consumer and producer surplus (the shaded area), resulting in an equilibrium supply of Q_1 at a marginal cost of c_1, given Q_1, the algorithmic steps are repeated to converge at the equilibrium solution (P^*, Q^*).

For a multi-market general equilibrium - the shifts in the demand for one good induce changes in the demand for other goods through general equilibrium income and cross-market price effects.

Equilibrium effects lead to the energy demand function $D_1 - D_1$ (do not present this level change in the figure) which is approximated locally at *b*. The iterative algorithm quickly converges as the decomposed energy sector is relatively small compared to the rest of the economy: The Marshallian demand approximation in the energy sector model then provides a precise local representation of the general equilibrium demand.

4. Implementation

Suppose a representative economic agent with two non-energy goods (x, y) and a set of four energy goods (*oil, gas, biomass and electricity)*. We begin with an algebraic characterization of the primal optimization setting. Then we provide a re-scheme of the model as a mixed complementarily problem. Next, we lay out the decomposition of the integrated top-down/bottom-up model and describe how such a model can be calibrated to base-year social accounts. Finally, we refer to a large-scale implementation of the decomposition (e.g., multi-region intertemporal general equilibrium) that combines a bottom-up representation of the energy supply sector with a top-down description of macroeconomic production and consumption (Manne et al., (2006).

4.1 The Integrated Model - Primal Optimization Setting

Energy goods are produced by a discrete number of technologies. Aggregate supply E_i of energy good *i* equals output z_{it} from all technologies *t* producing that energy good *i*:

$$E_i = \sum_t z_{it} \quad (6)$$

Consumer demand is modelled as budget-constrained utility maximization by a representative agent:

$$\max u(x_c, y_c, E_c)$$

$$\text{s.t. } p_x x_c + p_y y_c + \sum_i p_i^E E_i^c = M$$

Where, u denotes the utility from consumption of non-energy goods x_c and y_c as well as from the final energy consumption composite $E_c; p_x; p_y$ and p_i^E are the prices for non-energy and energy goods; E_i^c refers to the final consumption demand of energy good i; and M denotes the income of the representative household. Consumer preferences which trade off composite final energy E^c and non-energy goods at a constant elasticity of substitution (CES) are given as:

$$u(x_c, y_c, E^c == (\alpha E_c^{(1-\sigma^c)} + (1-\alpha)(x_c^{\theta^c} y_c^{(1-\theta^c)})^{1-\sigma^c})^{1/(1-\sigma^c)} \quad (7)$$

Where, σ^c is the substitution elasticity; α denotes the distribution parameter; and θ^c is the value share of x demand in the Cobb-Douglas xy - composite of final demand. Substitution possibilities across energy goods in final demand are characterized by a CES function:

$$E_0 = (\sum_i \beta_i E_i^{c\left(1-\sigma^{EC}\right)})^{1/(1-\sigma^{EC})} \quad (8)$$

Where, β_i is the distribution parameter; and σ^{EC} denotes the elasticity of substitution. Consumer income (M) is determined by wages w, earnings r on sector-specific capital and scarcity rents u_{it} on capacities of energy technology t producing energy good i:

$$M = \gamma_x \overline{K}_x + \gamma_y \overline{K}_y + w\overline{L} + \sum_{it} \mu_{it} \overline{z}_{it} \quad (9)$$

Where, $\overline{K}_y, \overline{K}_x$ denote sector-specific (fixed) capital; $\overline{L}$ is the fixed labour supply; and $\overline{z}_{it}$ denotes the capacity constraint on technology t producing energy good i. Goods x and y introduce intermediate demand to energy production and final consumption demand:

$$x = \sum_{it} \alpha_{it}^x z_{it} + x_c \quad (10)$$

$$y = \sum_{it} \alpha_{it}^y z_{it} + y_c \quad (11)$$

Where, $a_{it}^y (a_{it}^x)$ denote the (per-unit) input coefficient of non-energy input to the production of energy good i by technology t; z_{it} is the activity level of technology t delivering energy good i. Energy supplies are introduced as intermediate inputs into the production of non-energy goods and final demand. Furthermore, energy supplies serve as intermediate inputs to the production of other energy goods. The market clearance condition for energy good i is:

$$E_i = E_i^x + E_i^y + E_i^c + \sum_{i't} b_{ii't} z_{i't} \quad (12)$$

Where, $b_{ii't}$ is the input coefficient of energy good i into technology t producing energy good i'. The labour market is cleared by the real wage w:

$$L_x + L_y = \overline{L} \quad (13)$$

Likewise, rental rates γ_x and γ_y clear sector-specific capital markets:

$$K_x = \overline{K}_x \quad (14)$$

$$K_y = \overline{K}_y \quad (15)$$

Upper binds on energy sector technologies are realized through adjustment of technology-specific rents μ_{it}:

$$0 \le z_{it} \le \bar{z}_{it} \quad (16)$$

Production of no-energy goods x and y is based on profit maximization subject to technical constraints:

$$\max p_x x - wL_x + \sum_i p_i^E E_i^x \quad \text{s.t.} \quad x = f_x(\overline{K}_{x,}, L_x, g_x(E^x))$$

and

$$\max p_y y - wL_y + \sum_i p_i^E E_i^y \quad \text{s.t.} \quad y = f_y(\overline{K}_{y,}, L_y, g_y(E^y))$$

Three-level nested separable CES functions characterize trade-offs between primary factors and energy in the production of goods x and y. At the top level, energy composite is combined with a Cobb-Douglas aggregate in labour and capital subject to a constant elasticity of substitution:

$$f_i(\overline{K}_i, L_i, E_i) = \phi(\gamma_i E_i^{(1-\sigma_i)} + (1-\gamma_i)\overline{K}_i^{\theta} L_i^{(1-\theta_i)(1-\sigma_i)} \; i \in \{x, y\}$$

Where, ϕ_i is the efficiency parameter, γ_i is the distribution parameter, and σ_i is the elasticity of substitution.

At the lower level, energy inputs are combined (to a sector-specific energy input composite E_i) distinguishing substitutability differences between electricity, biomass, oil, and gas:

$$E_i = E_{ele,i}^{\theta_i^{ele}} (\delta_i E_{bms,i}^{(1-\sigma_i^E)} + (1-\delta_i)(E_{oil,i}^{\theta_i,oil} E_{gas,i}^{(1-\theta_i,gas}))^{(1-\sigma_i^E)})^{(1-\theta_i^{ele})/(1-\sigma_i^E)} \; i \in \{x, y\}$$

Where, θ_i^{ele} is the value share of electricity in the Cobb-Douglas energy composite demand of sector *i*; δ_i is the distribution parameter; θ_i^{oil} refers to the value share of oil in the Cobb- Douglas oil-gas composite; σ_i^E denotes the substitution elasticity between biomass and the oil & gas composite. Energy sector supplies are produced by profit-maximizing firms. The technology *t* that produces energy good *i* is then selected at a level which maximizes returns subject to capacity constraints:

$$\max z_{it}(p_i^E - p_x a_{it}^x - p_y a_{it}^y - \sum_{i'} p_i^E b_{i'it}) \quad \text{s.t.} \quad z_{it} \le \bar{z}_{it}$$

4.2 The Integrated Model – MCP

The model presented above omits a number of complications which arise in applied general equilibrium models. These might include multiple consumers with distinct preferences, taxes and incomes, knowledge spillover, or from the market side different structures like imperfect competition. In the absence of these features which typically violate integrability conditions, the integrated model can be solved as a conventional non-linear program by maximizing *u* subject to (6) through (16).

The optimization approach is, however, often too restrictive in terms of the model features which need to be included for concrete policy analysis. The complementarily format offers a flexible alternative to non-linear optimization as a mean of representing economic equilibrium models through "canonical" general equilibrium conditions (see conditions (1), (2), and (3)).

The algebraic representation begins from the dual cost minimization problems of the individual producers. For sectors *i = {x, y}* we have cost-minimizing unit energy costs given by:

$$p_i^E = \left(\frac{p_{ele}}{\theta_i^{ele}}\right)^{\theta_i^{ele}} \left\{ \delta_i \left(\frac{p_{bmas}}{\delta_i}\right)^{(1-\sigma_i^E)} + (1-\delta_i)\left[\left(\frac{p_{oil}}{(1-\delta_i)\theta_i^{oil}}\right)^{\theta_i^{oil}} \left(\frac{p_{gas}}{(1-\delta_i)(1-\theta_i^{oil})}\right)^{(1-\theta_i^{oil})}\right]^{(1-\sigma_i^E)} \right\}^{1/(1-\sigma_i^E)}$$

Unit profits functions for x and y are in turn given by:

$$\Pi_i = p_i - \frac{1}{\phi_i}\left[\gamma_i \left(\frac{p_i^E}{\gamma_i}\right)^{(1-\sigma_i)} + (1-\gamma_i)\left(\frac{\gamma_i}{\theta_i(1-\gamma_i)}\right)^{\theta_i(1-\sigma_i)} \left(\frac{w}{(1-\theta_i)(1-\gamma_i)}\right)^{(1-\theta_i)(1-\sigma_i)}\right]^{1/(1-\sigma_i)}$$

The unit cost of energy inputs to final demand are given by:

$$p_c^E = \left(\sum_i \beta_i \left(\frac{p_i^E}{\beta_i}\right)^{1-\sigma^{E\sigma}}\right)^{1/(1-\sigma^{EC})}$$

And the resulting cost of a unit of final consumption is:

$$p^c = \left[\alpha \left(\frac{p_c^E}{\alpha} \right)^{1-\sigma^c} + (1-\alpha) \left(\left(\frac{p_x}{\theta^c (1-\alpha)} \right)^{\theta^c} \left(\frac{p_y}{(1-\theta^c)(1-\alpha)} \right)^{(1-\theta^c)} \right)^{1-\sigma^c} \right]^{1/(1-\sigma^c)}$$

Finally, the unit profit associated with technology t for energy good *i = {oil, gas, biomass}* is:

$$\Pi_{it}^E = p_i^E - p_x a_{it}^x - p_y a_{it}^y - \sum_{i'} p_i^E b_{i'it} - \mu_{it}$$

Given the underlying functional forms, we observe that the complementarily conditions only will apply for the energy sector technologies and the shadow prices on the associated capacity constraints; all of the macro economic prices and quantities will be non-zero. According to Shepard's Lemma we have the following mixed complementarily problem:

Zero-profit conditions:

$$z_{it} \geq z_{it} \perp \mu_{it} \geq 0 \quad (17)$$

$$-\Pi_{it}^E \geq 0 \perp z_{it} \geq 0 \quad (18)$$

$$\Pi_x = 0 \quad (19)$$

$$\Pi_y = 0 \quad (20)$$

Market clearance conditions:

$$x = \sum_{it} \alpha_{it}^x z_{it} + c \frac{\partial \Pi_C}{\partial p_x} \quad (21)$$

$$y = \sum_{it} \alpha_{it}^y z_{it} + c \frac{\partial \Pi_C}{\partial p_y} \quad (22)$$

$$\overline{L} = x \frac{\partial \Pi_x}{\partial \Pi_w} + y \frac{\partial \Pi_y}{\partial \Pi_w} \quad (23)$$

$$\overline{K}_x = x \frac{\partial \Pi_x}{\partial \gamma_x} \quad (24)$$

$$\overline{K}_y = y \frac{\partial \Pi_y}{\partial \gamma_y} \quad (25)$$

$$\sum_t z_{it} - \sum_{i't} b_{ii't} z_{i't} = c \frac{\partial \Pi_c}{\partial p_i^E} + x \frac{\partial \Pi_x}{\partial p_i^E} + y \frac{\partial \Pi_y}{\partial p_i^E} \quad (26)$$

$$c = \frac{M}{p_c} \quad (27)$$

Income balance

$$M = \gamma_x \overline{K}_x + \gamma_y \overline{K}_y + w\overline{L} + \sum_{it} \mu_{it} \overline{z}_{it} \quad (28)$$

Activity Variables

c	Aggregate consumption;
x, y	Production of goods *x* and *y;*
E_i	Aggregate output of energy good *i* ;
z_{it}	Production by technology *t* for energy good *i;*
E_i^x, E_i^y	Demand for energy good *i* in sector *x* and *y;*
E_i^c	Final demand for energy good *I;*
L_x, L_y	Labor demand in goods *x* and *y;*

Price Variables

p_c	Price index of final consumption;
$p_{x,}p_y$	Non-energy goods x and y;
p_i^E	Energy prices for *i* = {*oil, gas, electricity*};
w	Wage rate;
γ_x, γ_y	Returns to non-energy capital;
μ_{it}	Energy sector rents;

Income Variables

M	Income of representative agent;

4.3 Decomposition

Decomposition strategy mainly requires the splitting of integrated model into a top-down model for the overall economy and a bottom-up model of the energy supply system, which might be a computable general equilibrium scheme. Within the top-down model, we treat net energy system net puts as exogenous. Energy supply activities are no longer endogenous and we can drop equations (17) and (18). Net energy supplies and inputs of non-energy goods to the energy system enter the top-down model as parameters.

Parameterized energy-sector net puts $\tilde{S}_i$ and inputs $\tilde{x}_E$ and $\tilde{y}_E$ are valued at market prices which implicitly include rents on specific energy resources (so we can drop these from the income constraint). The adjusted market clearance condition for energy goods within the top-down model is:

$$\tilde{S}_i = E_i^x + E_i^y + E_i^c \quad (29)$$

and the revised market clearance conditions for non-energy goods are:

$$x = \tilde{x}_E + c\frac{\partial \Pi_c}{\partial p_x} \quad (30)$$

and

$$y = \tilde{y}_E + c\frac{\partial \Pi_c}{\partial p_y} \quad (31)$$

The revised income balance (28) reads:

$$M = \gamma_x \overline{K}_x + \gamma_y \overline{K}_y + w\overline{L} + \sum_i p_i^E \tilde{S}_i - p_x \tilde{x}_E - p_y \tilde{y}_E \quad (32)$$

The bottom-up model can be represented as a quadratic programming problem - the sum of producer and consumer surplus is maximized subject to supply-demand balances for energy and resource bounds on technologies:

$$\max \sum_i \tilde{p}_i^E \left(1 + \frac{2\overline{S}_i - S_i}{2\varepsilon_i S_i}\right) - \tilde{p}_x x_E - \tilde{p}_y y_E \quad (33)$$

s.t.

$$S_i = \sum_t z_{it} - \sum_{i't} b_{ii't} z_{i't}$$

$$x_E = \sum_{it} \alpha_{it}^x z_{i't}$$

$$y_E = \sum_{it} \alpha_{it}^y z_{i't}$$

$$0 \le z_{it} \le \overline{z}_{it}$$

- Variables and Parameters in Decomposed Model

S_i Net supply of energy *i;*

x_E, y_E Aggregate demand for *x* and *y* as inputs to energy production;

$z_{i,t}$ Activity level of technology *t* producing energy good *i;*

Parameters

$\tilde{S}_i$ Reference level of demand or supply for energy good *i;*

$\tilde{p}_i^E$ Reference price of energy good *i* ;

$\tilde{p}_x, \tilde{p}_y$ Reference prices of non-energy goods *x* and *y;*

ε_i Demand elasticity for energy *i;*

4.4 Parameterization

According to King (2005), the model must be parameterized with economic data. The benchmark statistics are given in terms of a social accounting matrix (SAM), because their provided details of the energy demand structure in sectors *x* and *y* as well as in final demand - "*e*" summarizes total energy supplies by energy carrier *i* and non-energy inputs to energy production.

Base Year - Social Accounts Matrix

	x	*y*	*e*	*fd*
x	-			-
y		-	-	-
L	-	-		-
k	-	-		-
ele	-	-	-	-
oil	-	-	-	-
gas	-	-	-	-
bms	-	-	-	-

Key

x:*Energy intensive production*
y: *macro production*
e: *energy production*
fd:*final demand*
ele: *electricity*
oil: *oil*
gas: *natural gas*
bms: *biomass*

The generic procedure in order to aggregate the economy's energy supply side can be further detailed through a discrete representation of energy supply technologies thereby warranting consistency with the aggregate data.

First, is necessary to specify the desired number of technologies which are available for the generation of energy commodities. Second, randomly generate cost distribution for each technology thereby assigning a certain fraction of technologies as initially idle at benchmark prices. The cost structure of discrete technologies – fuel costs and non-energy input costs – is then again assigned randomly; capital earnings (i.e. scarcity rents on technological capacities, are determined as a residual) - if a specific technology is initially idle, the initial rents are obviously zero. Finally, relative capacities are randomly assigned and scaled such that net energy supply equals the given overall economic energy demand.

The decomposed integrated model is solved iteratively in its top-down and bottom-up sub-models. After an exogenous policy shock, we may first solve the top-down model. Next, we solve the bottom-up model taking into account the equilibrium prices of the top-down model. The solution values of the bottom-up model are subsequently used to update the quantities on energy system outputs and inputs which enter into the top-down model.

4.5 Large - Scale Implementation

To assess the performance for large-scale problems is necessary to implement a decomposition algorithm. The decomposition provides a convenient approach to solve the large-scale energy supply model for example (non-exclusive procedure) as quadratic programming problem without the explicit treatment of income effects. Within a single iteration of the decomposed solution process, output from the macroeconomic model characterizes demand for electric and non-electric energy by region and time period. The energy model then calculates the evolution of technologies which supply electric and non-electric energy, contingent on energy demands.

Energy demands are usually represented by linear demand functions (also non-linear specifications are possible) calibrated to the current solution of the macroeconomic top-down model.

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